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The effect of probabilities of arrivals with time in a bank

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Abstract— This paper deals with queueing theory and analysis of probability curves of pure birth model. Starting with historical back grounds and important concepts of queueing theory, we obtained a relation to find time "t" where we get highest probability for arrivals which follow Poission probability distribution.

Index Terms- arrivals, arrival rate, arrival distribution theorem, highest probability, Queues, poisson probability distribution, probability graph.

1 INTRODUCTION

A lot of our time is consumed by unproductive activities. Travelling has its own demerits one of them being wastage of time getting caught in traffic jams. A visit to the Post office or bank is very time consuming as huge crowds are waiting to be serviced. A simple shopping chore to a supermarket leads us to face long queues. In general we do not like to wait. People who are giving us service also do not like these delays because of loss of their business. These waits are happening due to the lack of service facility. To provide a solution to these problems we analyze queueing systems to understand the size of the queue, behavior of the customers in the queue, system capacity, arrival process, service availability, service process in the system. After analyzing the queueing system we can give suggestions to management to take good decisions.

A queue is a waiting line. Queueing theory is mathematical theory of waiting lines. The customers arriving at a queue may be calls, messages, persons, machines, tasks etc. we identify the unit demanding service, whether it is human or otherwise, as a customer. The unit providing service is known as server. For example (1) vehicles requiring service wait for their turn in a service center. (2) Patients arrive at a hospital for treatment. (3) Shoppers are face with long billing queues in super markets. (4) Passengers exhaust a lot of time from the time they enter the airport starting with baggage, security checks and boarding.

Queueing theory studies arrival process in to the system, waiting time in the queue, waiting time in the system and service process. And in general we observe the following type of behavior with the customer in the queue.

They are

Balking of Queue: Some customers decide not to join the queue due to their observation related to the long length of queue, in sufficient waiting space. This is called Balking.

Reneging of Queue: This is the about impatient customers. Customers after being in queue for some time, few customers become impatient and may leave the queue. This phenomenon is called as Reneging of Queue.

Jockeying of Queue: Jockeying is a phenomenon in which the customers move from one queue to another queue with hope that they will receive quicker service in the new position.

Important concepts in Queueing theory

Little law

One of the feet of queueing theory is the formula little law. This is

$$N = \lambda T$$

This formula applies to any system in equilibrium (steady state).

Where λ is the arrival rate

T is the average time a customer spends in the system

N is the average number of customers in the system

Little law can be applied to the queue itself.

I.e. $$N_q = \lambda T_q$$

Where λ is the arrival rate

T_q the average time a customer spends in the queue

N_q is the average number of customers in the queue

Classification of queuing systems

Input process

If the occurrence of arrivals and the offer of service strictly follow some schedule, a queue can be avoided. In practice this is not possible for all systems. Therefore the best way to describe the input process is by using random variables which we can define as "Number of arrivals during the time interval" or "The time interval between successive arrivals"

Service Process

Random variables are used to describe the service process which we can define as "service time" or "no of servers" when necessary.

Number of servers

Single or multiple servers

Queue length

1 to ∞

System capacity

Finite or Infinite

Queue discipline

This is the rule followed by server in accepting the customers to give service. The rules are

- FCFS (First come first served).
- LCFS (Last come first served).

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- Random selection (RS).
- Priority will be given to some customers.
- General discipline (GD).

Notation for describing all characteristics above of a queueing model was first suggested by David G Kendall in 1953.

- A/B/P/Q/R/Z

Where A indicates the distribution of inter arrival times
B denotes the distribution of the service times
P is the number of servers
Q is the capacity of the system
R denotes number of sources
Z refers to the service discipline

Examples of queueing systems that can be defined with this convention are M/M/1
M/D/n
G/G/n

Where M stands for Markov
D stands for deterministic
G stands for general

History In Telephone system we provide communication paths between pairs of customers on demand. The permanent communication path between two telephone sets would be expensive and impossible. So to build a communication path between a pair of customers, the telephone sets are provided a common pool, which is used by telephone set whenever required and returns back to pool after completing the call. So automatically calls experience delays when the server is busy. To reduce the delay we have to provide sufficient equipment. To study how much equipment must be provided to reduce the delay we have to analyze queue at the pool. In 1908 Copenhagen Telephone Company requested Agner K.Erlang to work on the holding times in a telephone switch. Erlang's task can be formulated as follows. What fraction of the incoming calls is lost because of the busy line at the telephone exchange? First we should know the inter arrival and service time distributions. After collecting data, Erlang verified that the Poisson process arrivals and exponentially distributed service were appropriate mathematical assumptions. He had found steady state probability that an arriving call is lost and the steady state probability that an arriving customer has to wait. Assuming that arrival rate is λ, service rate is μ and

$$\rho = \frac{\lambda}{\mu}$$

he derived formulae for loss and deley.

(1) The probability that an arriving call is lost (which is known as Erlang B-formula or loss formula).

$$P_n = \frac{\frac{\rho}{n!}}{\sum \frac{\rho^k}{k!}} = B(n,\rho)$$

(2) The probability that an arriving has to wait (which is known as Erlang C-formula or deley formula).

$$P_n = \frac{n}{n-\rho(1-B(n,\rho))} B(n,\rho)$$

Erlang's paper "On the rational determination of number of circuits" deals with the calculation of the optimum number of channels so as to reduce the probability of loss in the system. Whole theory started with a congestion problem in teletraffic. The application of queueing theory scattered many areas. It include not only tele-communications but also traffic control, hospitals, military, call-centers, supermarkets, computer science, engineering, management science and many other areas.

Arrival Distribution theorem

If the arrivals are completely random then the probability distribution of number of arrivals in a fixed time interval follows a Poisson distribution

$$P_n(t) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$$

This equation describes the probability of seeing n arrivals in a period from 0 to t.
t is used to define the interval 0 to t.
n is the total number of arrivals in the interval 0 to t.
λ is the total average arrival rate in arrivals per second.

A short survey conducted on Andhra Bank, J.N.T.U.H., and Hyderabad. The bank is with an average of 3 customers arriving every 100 seconds (0.03 arrivals/second arrival rate). Let X is the Random variable defined as "Number of arrivals at the bank during the time interval 0 to t" which describes the input process at the bank. Since all the customers are independent i.e. their decision to enter the bank is independent leads us to consider arrivals are completely random. Then the probability distribution of number of arrivals in a fixed time interval follows a Poisson distribution

$$P_n(t) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$$

With the arrival rate 0.03 arrivals/second and by using Poisson probability distribution we find the probability of number of arrivals during the time interval 0 to t. The probabilities of arriving one person is calculated through the formula

$$P_1(t) = \frac{e^{-\lambda t}(\lambda t)^1}{1!}$$

Noting the probabilities of arriving 2, 3 and 4 persons during the time interval 0 to t by using the formulae

$$P_2(t) = \frac{e^{-2\lambda t}(\lambda t)^2}{2!}$$

$$P_3(t) = \frac{e^{-3\lambda t}(\lambda t)^3}{3!}$$

$$P_4(t) = \frac{e^{-4\lambda t}(\lambda t)^4}{4!}$$

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If we observe the bank for a period of 1 second, the probability of arriving one customer in 0 to 1 seconds time interval is 0.029. If we observe the bank for 2 seconds, the probability of arriving one customer in 0 to 2 seconds time interval is 0.056. If we observe the bank for 5 seconds the probability of arriving one customer in 0 to 5 seconds time interval is 0.129. Up to 180 seconds the probabilities of arriving n persons is calculated.

$P_1(1) = 0.029113366$ $P_2(1) = 0.0004367$
$P_1(2) = 0.056505872$ $P_2(2) = 0.0016951$
$P_1(3) = 0.082253806$ $P_2(3) = 0.003701$
$P_1(4) = 0.09048$ $P_2(4) = 0.006385$
$P_1(5) = 0.129106196$ $P_2 5) = 0.0968296$
$P_3(1) = 0.0000043$ $P_4(1) = 0.00000003$
$P_3(2) = 0.0000339$ $P_4(2) = 0.00000050$
$P_3(3) = 0.0001110$ $P_4(3) = 0.00000249$
$P_3(4) = 0.0002583$ $P_4(4) = 0.00000766$
$P_3(5) = 0.0002583$ $P_4(5) = 0.00001815$

Remaining values are tabulated.

Table - 1 to Table - 4 gives statistics of probabilities of arrivals of n number of persons at time t.

1

time	$P_1(t)$	time	$P_1(t)$
1	0.029113366	95	0.16485631
2	0.056505872	100	0.149361
3	0.082253806	105	0.134984
4	0.09048	110	0.121714
5	0.129106196	115	0.109522
10	0.222245466	120	0.098365
15	0.286932668	125	0.08819
20	0.329286981	130	0.078943
25	0.354274194	135	0.07056
30	0.365912693	140	0.06298
31	0.36693495	145	0.0561446
32	0.36757717	150	0.049990
33	**0.367860924**	155	0.039502
34	0.367806839	160	0.039502
35	0.367434636	165	0.035062
40	0.361433054	170	0.0310093
45	0.349974351	175	0.027549
50	0.3346	180	0.024389
55	0.316882349		
60	0.297537998		
65	0.2774344		
70	0.257145		
75	0.2371275		
80	0.216264		
85	0.199104		
90	0.18145488		

2

time	$P_2(t)$	time	$P_2(t)$
1	0.0004367	95	0.2349189
2	0.0016951	100	0.2240415
3	0.003701	105	0.21259948
4	0.006385	110	0.2008284
5	0.0968296	115	0.188922431
10	0.0607436	120	0.17705304
15	0.0645598	125	0.165304687
20	0.0987860	130	0.1539252
25	0.1328530	135	0.14286577
30	0.1646607	140	0.1322559
35	0.1929031	145	0.12205012
40	0.2168598	150	0.1124685
45	0.2362326	155	0.1033555
50	0.2510214	160	0.0947980
55	0.2614279	165	0.0867756
60	0.2677841	170	0.0792784
65	0.2704338	175	0.0723171
66	0.270643318	180	0.0658432
67	**0.270663822**		
68	0.270563741		
70	0.27000225		
75	0.2667684		
80	0.2612448		
85	0.2538576		
90	0.2449622		

3

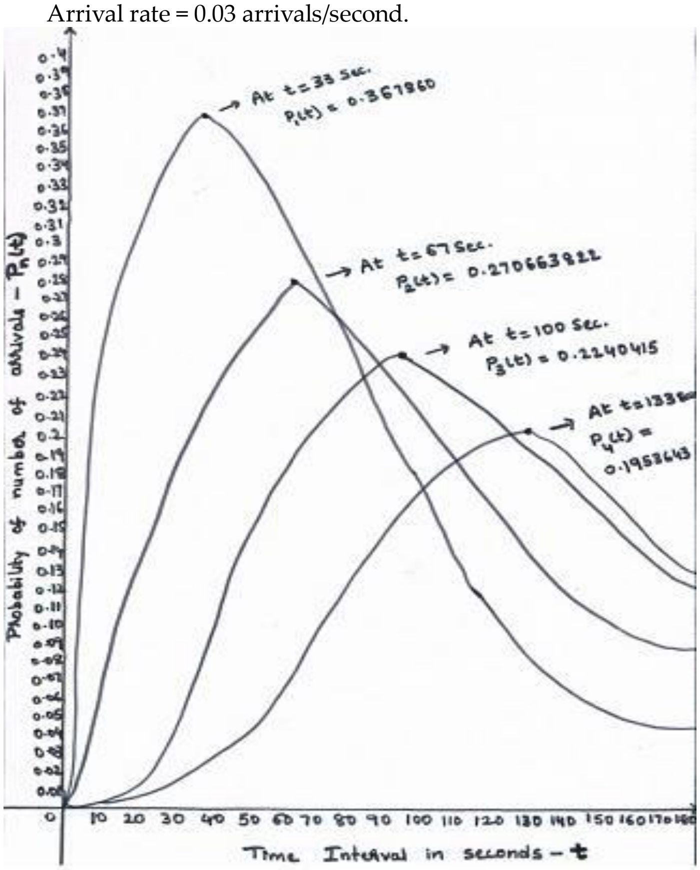
Time	$P_3(t)$	Time	$P_3(t)$
1	0.0000043	96	0.223490004
2	0.0000339	97	0.223733375
3	0.0001110	98	0.223905604
4	0.0002583	99	0.224007978
5	0.0004841	**100**	**0.2240415**
10	0.003333	101	0.22400826
15	0.009683	105	0.223229
20	0.019757	110	0. 229113
25	0.033213	115	0.217260795
30	0.049398	120	0.212463648
35	0.067516	125	0.206630859
40	0.086743	130	0.20010276
45	0.106304	135	0.192868796
50	0.125510	140	0.18515826
55	0.143785	145	0.176972681
60	0.160670	150	0.16870275
65	0.175781	155	0.160201102
70	0.189001	160	0.151676928
75	0.200076	165	0.143179746
80	0.208995	170	0.134773416
85	0.215778	175	0.126554941
90	0.220466	180	0.118517904
95	0.223172997		

4

time	$P_4(t)$	time	$P_4(t)$
1	0.00000003	95	0.1590107605
2	0.00000050	100	0.168031125
3	0.00000249	105	0.175793199

4	0.00000766	110	0.182251845
5	0.00001815	115	0.187387436
10	0.0000750	120	0.191217283
15	0.0010894	125	0.193716430
20	0.0029635	130	0.195100191
25	0.0062274	132	0.195327483
30	0.0111145	133	0.195364368
35	0.0177229	134	0.1953357079
40	0.0260231	135	0.192868796
45	0.0358778	140	0.18515826
50	0.0470665	145	0.17697268
55	0.0593162	150	0.16870275
60	0.0723017	155	0.16020110
65	0.0856936	160	0.15167692
70	0.0992258	165	0.14317974
75	0.1125429	170	0.13477341
80	0.1253975	175	0.12655494
85	0.1375593	180	0.11851790
90	0.1488145		

Graph (Probability graph)
Probability that number of customers arrive during a specified interval.
Arrival rate = 3 customers arriving every 100 seconds.
Scale: Time interval in seconds (0 to 180seconds)
$P_n(t)$ is 0 to 1
Arrival rate = 0.03 arrivals/second.



A graph is drawn between time and probability of n arrival at time t. Time in seconds is taken on the horizontal line, $P_n(t)$ is taken on the vertical line. Curve - 1 (c-1) is derived by taking n = 1 in the Poisson arrival distribution. As well remaining curves c-2, c-3, c-4 have been derived by taking n = 2, n = 3, n = 4 respectively. By observing the graph, we would note that each curve peaks at a particular point and then they start to descend. After descending to a certain point they run parallel to the time axis. We can observe that highest probability for curve - 1 is at t = 33seconds. This indicates probability of arrival of one person to the bank from 0 to 33seconds is high.

Let α denote the time at which Curve-1 takes highest point. Here α=33 seconds. Curve -2 got highest point at t = 67 seconds. Third curve got highest point at time t = 100 seconds. Forth curve got highest point at time t = 133 seconds.
For the curve-1 at t=33seconds, the crest point $P_1(33)$= 0.367860924.
For the curve-2 at t=67seconds, the crest point $P_2(67)$= 0.270663822.
For the curve-3 at t=100seconds, the crest point $P_3(100)$= 0.2240415.
For the curve-4 at t=133 seconds the crest point $P_4(133)$= 0.1953643.
Developing appropriate formula for t at which we get highest probability by use of crest points. For n = 1 we have crest point at t = 33 seconds. That is α = 33 seconds.

For n = 2 n(33)+1 or n(33)+2
2(33)+1 or 2(33)+2
67 or 68

The maximum probability for curve - 2 is attained at 67 seconds.

For n = 3 n(33)+1 or n(33)+2
3(33)+1 or n(33)+2
100 or 101

The maximum probability for curve - 3 is attained at 100 seconds.

For n = 4 n(33)+1 or n(33)+2
4(33)+1 or n(33)+2
133 or 134

The maximum probability for 4th curve is attained at 133 seconds.

For n = 5 n(33)+1 or n(33)+2
5(33)+1 or n(33)+2
166 or 167

In the same manner with n = 6
6(33)+1 or 6(33)+2

At one of these points we can get highest probability. The above calculations can be generalized and is indicated by the following formula.

n(α)+1 or n(α)+2, n is an integer and n≥2,
α is the time where we got crest point for curve -1

Conclusions

This paper has surveyed the use of queueing theory for the analysis of different curves. From the work surveyed above we have found arrival rate at the bank is 0.03 arrivals per second. We Considered the Random variable as "Number of arrivals during the time interval 0 to t" which follows Poission probability distribution. By using Arrival distribution theorem i.e. If the arrivals are completely random then the probability distribution of number of arrivals follows Poisson probability distribution

$$P_n(t) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}.$$

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We calculated probabilities of arrival of n = 1,2,3,4 persons during time interval 0 to t. A graph was drawn between the time and probabilities of n arrivals where n = 1, 2, 3, 4, from which curves c-1, c-2, c-3 and c-4 were obtained. By observing the graph, we would note that each curve peaks at particular point and then they starts to descend. After getting the peak point of curve-1 we assumed that point as α. For the remaining curves from the observations we got peak points at particular time. From the above calculations we developed two equations. Those equations are (Time) t = $n(\alpha)+1$ or $n(\alpha)+2$, n is an integer and $n \geq 2$, α is the time where we got crest point for curve-1.These equations assist us in deriving time where curve takes crest point. We observe that for fixed n, if time increases probability of arriving n persons increases. i.e. $t \propto P_n(t)$ where n is constant. For fixed t, as n increases probability of arriving n persons decreases.

$$\text{I.e. } n \propto \frac{1}{Pn(t)}$$

where t is a constant. As n increases highest probability value (crest point value) decreases.

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